

Roll No.

Paper Code – BCA 105 (N)

BCA 1st Year (Semester-I) EXAMINATION, 2023-24

MATHEMATICS-I

PAPER-V

Time : Two Hours]

[Maximum Marks : 75

Note- This paper consists of three Section A, B and C.

Carefully read the instructions of each Section in solving the question paper. Candidates have to write their answers in the given answer-copy only. No separate answer-copy (B Copy) will be provided.

Section-A

(Short Answer Type Questions)

Note- All questions are compulsory. Answer the following questions as short answer type questions. Each question carries 5 marks.

1. (A) If $A = \begin{bmatrix} 1 & 2 & 10 \\ 4 & 10 & -2 \\ 10 & -2 & 1 \end{bmatrix}$, Find adjoint of matrix A.

- (B) Find value of c using Lagrange's mean value theorem for $f(x) = x^4$ in interval $[1, 2]$.

- (C) Using properties of the determinant, show

$$\text{that } \begin{vmatrix} a+x & a-x & a-x \\ a & x & a-x \\ a+x & a-x & a-x \end{vmatrix} = 0.$$

- (D) Evaluate $\lim_{x \rightarrow 3} \frac{x^4 - 3^4}{x - 3}$.

- (E) Find the angle between $\vec{a} = 2i + 2j - k$ and $\vec{b} = 6i + 3j + 2k$.

- (F) Find the volume of parallelepiped with adjacent vertices vectors $\vec{a} = 3i + 2j - k$, $\vec{b} = i + j - k$, $\vec{c} = 2i - k$.

- (G) If $y = x \log x$, then find $\frac{dy}{dx}$.

- (H) Evaluate $\int \tan^{-1} x \, dx$.

- (I) Find the value of λ which makes the vectors $\lambda \hat{i} + 6\hat{j} + 2\hat{k}$ and $-\hat{i} + 2\hat{j} + \lambda \hat{k}$, perpendicular.

Section-B

(Long Answer Type Questions)

Note- This section contains four questions from which **one** question is to be answered as long question. Each question carries 15 marks.

2. (a) Find the differential coefficient of $(x)^x$.

- (b) Evaluate $\lim_{x \rightarrow 0} \frac{x \sin \frac{1}{x}}{e^x}$.

- (c) Discuss the continuity of $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.

(Or)

3. (a) Using properties of determinants find

$$\begin{vmatrix} a+b+c & a & b \\ c & a+b+c & b \\ c & a & a+b+c \end{vmatrix}.$$

- (b) Find Maclaurin's series expansion of function

$\tan^{-1} x$ about the point $x = 0$.

- (c) Discuss the extrema of function $f(x) = 2x^3 - 21x^2 + 36x - 20$.

(Or)

4. (a) Write statement of Caley Hamilton theorem and find characteristic equation of matrix

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}.$$

- (b) Verify statement of Caley Hamilton Theorem for matrix A given in part a.

- (c) If matrix A is invertible, then find A^{-1} .

(Or)

5. (a) Find the second order derivative of $\frac{x^2}{(x-a)(x-b)}$.

- (b) Find the inverse of matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 6 & 2 \\ -3 & 2 & -1 \end{bmatrix}$

using Adjoint method.

- (c) Evaluate the limit $\lim_{x \rightarrow \infty} \left(\frac{\log x}{x} \right)$.

Section-C

(Long Answer Type Questions)

Note- This section contains four questions from which **one** question is to be answered as long question. Each question carries 15 marks.

6. (a) Write the statement of Taylor's theorem and expand the function $f(x) = \log(\sec x)$.

(b) Evaluate indefinite integral $\int \frac{1}{1+\cos x} dx$.

(Or)

7. (a) Find the unit vector perpendicular to both the vectors $\vec{a} = 3\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

(b) Evaluate integral by parts $\int x^2 \cos x dx$.

(Or)

8. (a) If $\sin y = x \sin(b + y)$ find $\frac{dy}{dx}$.

(b) Evaluate integral $\int \frac{x^3}{(x-1)(x-2)^2} dx$ by using partial fraction method.

(Or)

9. (a) Find the work done by a force of magnitude 3 units acting in the direction $3\hat{i} - 2\hat{j} + 6\hat{k}$ acting on a particle, which is displaced from the point $A(2, -3, -1)$ to point $B(4, 3, 1)$.
- (b) Verify scalar triple product for any three vectors.

....